**MAS-003-001308** Seat No. ________

**B. Sc. (Sem. III) (CBCS) Examination**

**October / November – 2016**

**Mathematics : 301(A)**

*[Linear Algebra, Calculus & Theory of Equation]*

*(New Course)*

**Faculty Code : 003**
**Subject Code : 001308**

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

**1** Answer the following objectives : **20**

(1) Define linear span of vector set.

(2) Define linear dependent vector set.

(3) Define Range and Kernal of linear transformation.

(4) $T: R^3 \to R^2$; $T(e_1)=(1,1)$, $T(e_1+e_2)=(1,0)$, $T(e_1+e_2+e_3)=(1,-1)$ then $T(e_3)=$_____.

(5) Define Eigen value.

(6) State P-series test.

(7) Give Newton - Raphson formula to find approximate root of $f(x)=0$.

(8) Find $N_T$ for $T: R^3 \to R^2$, $T(x,y,z)=(x-y+z, x+y-z)$.

(9) $T: U \to V$ then $R_T$ is subspace of U is, True of False ?

(10) State D'Alembert's ratio test.

(11) Give unite length interval in which possitive root of the equation $x^3-2x-5=0$ lies.

(12) What is length of the tangent ? Define.

(13) Define Radius of Curvature.

(14) Give formula to find radius of curvature of curve $y=f(x)$.

(15) Define point of inflexion.

(16) Give condition for the curve $y = f(x)$ is convex upwards at $x = c$.

(17) Define multiple point.

(18) Find radius of curvature at origin for

$x^3 - 2x^2y - 4y^3 + 5x^2 + 7y^2 - 8y = 0$

(19) Define node.

(20) Find radius of curvature of the curve $y = \log x$ at $(1, 0)$.

**2** (a) Answer any **three** : **6**

(1) Check whether $\{(1,1,-1), (1,0,1), (1,1,0)\}$ is linearly dependent or not.

(2) Prove that $W = \{(a,b,c) / 2a + 5b - c = 0\}$ is subspace of $R^3$.

(3) Find $R_T$ for, $T : R^2 \to R^3$, $T(x, y) = (x, x+y, y)$.

(4) Prove that composition of two linear transformation is also linear transformation.

(5) Show that the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots$ is convergent.

(6) State D'Alembert's Ratio test and Raabe's test.

(b) Answer any **three** : **9**

(1) Prove that intersection of two subspaces of v is also subspace.

(2) Expand set $\{(1,1,1), (0,1,1)\}$ to form a base of vector space $R^3$.

(3) Verify that $T : R^4 \to R^2$,

$T(x,y,z,w) = (x - y, y + z + w) \forall (x,y,z,w) \in R^4$ is linear transformation or not.

(4) $T : U \to V$ be any linear transformation then prove that $R_T$ is subspace of V and $N_T$ is subspace of U.

(5) Test the convergence of

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots\ldots\ldots.$$

(6) Test the convergence of $\frac{1}{1+2} + \frac{1}{1+2^2} + \frac{1}{1+2^3} + \ldots\ldots\ldots$.

(c) Answer any two : **10**

(1) $V=\{(x,y)/x>0, y>0, x,y\in R\}$ for $\alpha\in R$ and $(a,b),(c,d)\in V$ ; $(a,b)+(c,d)=(ac,bd)$ and $\alpha(a,b)=\left(a^{\alpha},b^{\alpha}\right)$ check whether V is vector space or not.

(2) Prove that $\left\{1, x, x^2+x, x^3+3x^2+2x\right\}$ is base of space $P_3(R)$.

(3) Vector $v_k$ $(1\le k\le n)$ of set $\{v_1, v_2, \ldots\ldots, v_n\}$ is linear combination of remaining vectors then prove that $s_p\{v_1, v_2, \ldots\ldots, v_n\}=s_p\{v_1, v_2, \ldots\ldots, v_{k-1}, v_{k+1}, \ldots\ldots, v_n\}$.

(4) Let $T:R^3\to R^2$, $T(a,b,c)=(2a+b-c, 3a-2b+4c)$ $\forall(a,b,c)\in R^3$ and $B_1=\{(1,1,1),(1,1,0),(1,0,0)\}$ $B_2=\{(1,3),(1,4)\}$ then find $[T:B_1,B_2]$.

(5) Examine the convergence of $1+\frac{1}{2}+\frac{1\cdot 3}{2\cdot 4}+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}+\ldots\ldots\ldots$.

**3** (a) Answer any **three** : **6**

(1) Derive formula to find $\frac{1}{N}$ using Newton-Raphson formula.

(2) Find radius of curvature for curve $S=c\log(\sec\psi)$.

(3) Find radius of curvature at the origin for $x^2y+xy^2+xy+y^2-3x=0$.

(4) Find radius of curvature of $y=4\sin x-\sin 2x$ at $x=\frac{\pi}{2}$.

(5) Find slope of oblique asymtotes of $y=\frac{x^2+2x-1}{x}$.

(6) Find the position and nature of the double point on $x^3+y^3-3xy=0$.

(b) Answer any **three** : **9**

(1) Explain graphical method to find solution of given equation.

(2) Derive formula to find $p^{th}$ root of given number using Newton-Raphson's method.

(3) Derive convergence criterion of root for Newton-Raphson's method.

(4) Prove that $y = e^x$ is concave upward everywhere and $y = \log x$ is convex everywhere.

(5) Find all the asymtotes of $xy^2 = 4a^2(2a - x)$.

(6) Find multiple points on the curve $x^3 + y^3 - 3x^2 - 3xy + 3x + 3y - 1 = 0$.

(c) Answer any **two** : **10**

(1) If $y = mx + c$ is an asymptote to the curve $y = f(x)$, prove that, $m = \lim_{x \to \pm\infty} \left(\frac{y}{x}\right)$ and $c = \lim_{x \to \pm\infty} (y - mx)$.

(2) Find radius of curvature at any point $(x, y)$ on the curve $y^2 = 4ax$.

(3) Derive formula to find radius of curvature for equation of the form $x = f(t)$, $y = g(t)$.

(4) Derive formula to find approximate root of $f(x) = 0$ using false possition method.

(5) Explain Bisection method to find approximate root of $f(x) = 0$.